THE SEVENTH TO TENTH GRADES A UNIT IN MATHEMATICS

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Since this paper is intended for educators interested in public-school work, it may be proper first to define the topic. This may be done best by giving an explanation of present conditions in our elementary and secondary courses.

The present course is: 7th and 8th grades, 1 unit; 9th grade, 1 unit; 10th grade, 1 unit. The paper is written to call attention to the defects of the course as now given and to raise the question: How may these defects be remedied?

For the past five years mathematics associations have taken up the cry for better mathematics teaching. A Professor Perry of England offered the first remedy by suggesting the teaching of physics and chemistry in algebra and the use of many measurements and experiments in the mathematics class. As a result, the disciples of this movement went up and down the land demanding correlation. But where are those disciples? You can't find them now. There was a fundamental error in the scheme. The material given the pupil to *use* in mathematics was beyond him, often beyond his comprehension. The result was, the student was so confused by the mass of new facts and new surroundings that he got no science and no mathematics.

We all teach some physics notions—it's the fashion—and a little such is well. But the teacher who tries to lead his pupil to discover and develop physics laws which cost the life-endeavor of some of the greatest intellects this world has known and to discover the equations governing those laws, certainly harms the pupil, the subject, and the cause of education.

There is a relation, a simple one, one that is easily within the grasp of every pupil, that has been entirely overlooked. With Professor Perry, I firmly believe that mathematics should not be

set off in compartments. However, I am of the opinion that the articulation must be made with *known subjects* rather than with *unknown subjects.*

In the grammar school we teach arithmetic—just arithmetic—such as has been taught in grades IV and V. So far as my observation goes there is little or no thought-work done, no development of principles, no application made. In the ninth grade we teach algebra—just algebra. No application is made; the arithmetic does not help the algebra nor does the algebra help the arithmetic; geometry is not anticipated. In grade X we teach geometry. Little or no use of algebra is made, the equation is neglected, and pure reason introduced. Arithmetic is too far in the past to get even a hearing.

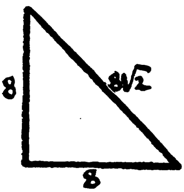
Such teaching during these four years is, to my mind, sheer waste of time. A few illustrations may strengthen the assertion. Your seventh-grade pupil, eighth-grade pupil, ninth-grade pupil, tenth-grade pupil knows nothing of the constitution of number, of the relation of numbers, of the use of number. He knows a few combinations of numbers. He knows a few rules and no *reasons why.* He may know $6 \cdot 8$ but when asked for $6 \cdot 18$ he frantically grasps a pencil or crayon. He may know the square of 32 but $16 \cdot 64$ is impossible without pencil and paper. He knows that $a^2 \cdot a^3 = a^5$ but he cannot tell you what power of 3 is $9 \cdot 27$. He will tell you that the square on the hypothenuse equals the sum of the squares on the two legs of the triangle but he cannot tell you whether the diagonal of a square whose side is 8 is 14, 15, or 16, even though he may possibly know that diagonal to be $8\sqrt{2}$. To multiply a fraction he is not quite sure whether to multiply the numerator or the denominator of the fraction by an integer, so to make certain that he does all that he should do he multiplies both. He knows the square of $a+b$ but he cannot tell you the square of $30\frac{1}{4}$. None of these defects is the fault of the child. They are chargeable directly to the teacher and to the courses offered.

Now for the remedy. A closer knitting together of arithmetic, algebra, and geometry, an articulation, if you please, of these subjects, and an understanding of principles rather than of rules will settle these troubles. For example, when your grammar-

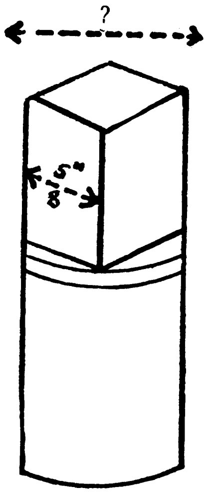
school pupil knows that $c(a+b)=ac+bc$, he should also know that $6 \cdot 18 = 6(10+8)$ and that $16 \cdot 17$ is also the product of a binomial and a monomial, namely $16(10+7)$. Mental arithmetic, rapid and accurate, is then possible. When your pupil knows how to write the product of $16 \cdot 17$, $(3x-17)(5x+16)$ needs no pencil and paper. You teach $32 \cdot 32 = 1024$, but do you teach $2^5=32$, $2^4=16$, $2^6=64$? You teach $(a^5)^2=a^{10}$, also $a^4 \cdot a^6 = a^{10}$. Do you teach $(2^5)^2 = 2^{10} = (32)^2$ and $2^4 \cdot 2^6 = 2^{10} = (32)^2 = 1024$?

$a^2 \cdot a^3 = a^5$. Does that mean to your pupil $3^2 \cdot 3^3 = 3^5$? That is, is $9 \cdot 27 = 3^5$?

$(a^2)^3 = a^6$ and $\sqrt{a^6} = a^3$. Does 9^3 mean 3^6, and does $\sqrt{9^3} = \sqrt{3^6} = 3^3 = 27$?

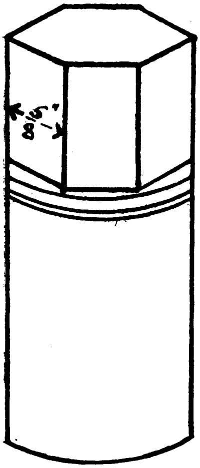


The side of a square is 8. The diagonal is $8\sqrt{2}$. Does your pupil have *any* idea of how long a line $8\sqrt{2}$ is? Does he know $8 \cdot 14$? Does he know that $8\sqrt{2} = 8(1.4+)$?



The stock for milling machine purposes is usually cylindrical. Try your pupil on this one: A piece of iron is to be milled so that one end is a square $\frac{5}{8}''$ on a side. What size stock must be selected? (See diagram.)

The problem is simply the application of the relation between the diagonal of a square and its side.



Now put the same question to the pupil making the end hexagonal, $\frac{5}{8}''$ on one side. Have you taught him why multiplying the numerator of a fraction multiplies the fraction or did he just learn the rule? Does he know that a mixed number is a binomial and that there are two kinds—the $a+b$ and the $a-b$—or does he square $30\frac{1}{4}$ by reducing to $\frac{121}{4}$ and squaring both numerator and denominator? I would suggest that the square of a mixed number be obtained mentally by means of, say,

$$(30\tfrac{1}{4})^2 = (30+\tfrac{1}{4})^2 = (900+15+\tfrac{1}{16}).$$

In reducing fractions to the lowest common multiple do you permit your pupils to follow this process:

$$\frac{5}{48}+\frac{11}{36}+\frac{8}{45}+\frac{7}{54}=\frac{\qquad\qquad}{2160}\ ?$$

$$\begin{array}{r|rrrr}
2 & 48 & 36 & 45 & 54 \\ \hline
2 & 24 & 18 & 45 & 27 \\ \hline
3 & 12 & 9 & 45 & 27 \\ \hline
3 & 4 & 3 & 15 & 9 \\ \hline
 & 4 & 1 & 5 & 3
\end{array}$$

Or would you combine algebra and arithmetic in this manner?

$48 = 2^4\cdot 3$
$36 = 3^2\cdot 2^2$
$45 = 3^2\cdot 5$
$54 = 3^3\cdot 2$

Select for your L. C. M. each factor which appears, giving to each the highest exponent appearing in any number concerned, i. e., $2^4\cdot 3^3\cdot 5$.

$2^4\cdot 3^3\cdot 5 \div 48$ is $2^4\cdot 3^3\cdot 5 \div 2^4\cdot 3 = 3^2\cdot 5$. The first numerator is then $3^2\cdot 5^2$. Similarly the second numerator is $2^2\cdot 3\cdot 5\cdot 11$. The third numerator, $2^4\cdot 3\cdot 8$ or $2^7\cdot 3$. The fourth numerator, $2^3\cdot 5\cdot 7$.

Then $\frac{5}{48}+\frac{11}{36}+\frac{8}{45}+\frac{7}{54}=\frac{3^2\cdot 5^2+2^2\cdot 5\cdot 11\cdot 3+2^7\cdot 3+2^3\cdot 5\cdot 7}{2^4\cdot 3^3\cdot 5}$

$$=\frac{225+660+384+280}{2160}$$

$$=\frac{200+600+300+200+20+60+80+80+5+4}{2160},$$

all of which may be performed mentally.

Such a course will not only help the arithmetic but will be of infinite service in the manipulation of algebraic fractions and in the reduction of surd polynomials: e. g.,

$$\sqrt{(a^2-a-12)(a^2+2a-3)(a^2-5a+4)}.$$

A similar articulation and application of subjects should extend through algebra and geometry. It may necessitate some rearrangement of courses in both grammar school and high school. But is it not worth while? Will it not teach the pupil to think? Will it not give him some opportunity to estimate his answer and to know within some limit at least how nearly right he is? Will such articulation not bridge over the chasm between grammar school and high school and save some of that wholesale slaughter that now occurs in the freshman year?

It is my opinion that such course can be carefully and successfully worked out. The mathematics for college, for business, for scientific lines will then be built on the foundation laid in the third to sixth grades, will be one continuous course, the various phases of secondary mathematics being brought in naturally and as rapidly as the pupil can assimilate the material. Usable mathematics can be taught and that is the only kind of mathematics that belongs to the pupil.